A. Galli et al., International Journal of Ecodynamics. Vol. 2, No. 4 (2007) 250–257

AN EXPLORATION OF THE MATHEMATICS BEHIND THE ECOLOGICAL FOOTPRINT

A. GALLI[1,2], J. KITZES[2], P. WERMER[2], M. WACKERNAGEL[2], V. NICCOLUCCI[1] & E. TIEZZI[1]
[1]Department of Chemical and Biosystems Sciences, University of Siena, Siena, Italy.
[2]Global Footprint Network, Oakland, CA, USA.

ABSTRACT

Introduced in the early 1990s, the ecological footprint has become a well-known and widespread environmental accounting tool. It measures human demand on nature and compares this to the availability of regenerative capacity on the planet. The method expresses human demand in terms of *global* hectares – i.e. biologically productive hectares with world-average productivity necessary for resource production and waste assimilation. Almost 15 years of research application and methodological advancements have made the ecological footprint an increasingly robust theoretical framework, and it continues to be refined. This article documents the most updated footprint methodology and focuses on the mathematics that supports footprint and biocapacity accounts, as well as its underlying factors such as *equivalence* and *yield* factors. To clarify the meaning and the usefulness of footprint and biocapacity reported in terms of *global* hectares, an in-depth description of the units of measure is presented. Finally, the different research questions that emerge when reporting data in *nation-specific* hectares as opposed to *global* hectares are investigated.

Keywords: actual hectares, biocapacity, ecological footprint, equivalence factors, global hectares, method, yield factors.

1 INTRODUCTION

Ecological footprint analysis was first introduced in the early 1990s by Mathis Wackernagel and William Rees [1, 2] as an environmental accounting tool to account for demand on (ecological footprint) and supply of (biocapacity) renewable natural capital. It provides an answer to the research question: how much of the regenerative capacity of the planet is occupied by human activities?

The demand side of this accounting framework is defined as human use of the annual regenerative capacity of the biosphere. This is expressed in mutually exclusive hectares of biologically productive land or sea area that are required to renew the resource throughput and absorb the waste production of a defined population in a given year. Prevailing technology, resource management, different consumption categories and land use areas are all taken into account while conducting the calculations [3–7].

While the footprint shows the demand on nature, the biocapacity tracks the supply side of the equation, and is therefore defined as the rate of resource supply and waste disposal that can be sustained in a given territory (or at the global scale) under prevailing technology and management schemes [8, 9].

Both footprint and biocapacity are usually expressed in terms of a common unit, the *global* hectare (*g*ha), which is one hectare of land or water normalized to have the world-average productivity of all biologically productive land and water in a given year [10].

Since the surface of the Earth is finite, the availability of biologically productive area and the annual amount of resource production and waste disposal are finite as well. Therefore, the use of an area as a measure of life supporting natural capital was chosen to reflect the fact that many basic ecosystem services and ecological resources are provided by surfaces where photosynthesis takes place [11]. This shows how humanity is constrained by nature's negentropic capacity to transform low-quality solar energy into high-quality chemical energy and living matter, available for all living species [12–14]. (H.T. Odum's hierarchical scale for energy quality ranking is used here.)

ISSN: 1743-5242 (paper format), ISSN: 1743-5250 (online), http://journals.witpress.com
DOI: 10.2495/ECO-V2-N4-250-257

To provide a quantitative and qualitative answer to this research question, the ecological footprint accounting method is based on the following main assumptions [11]:

- The annual amount of resources consumed and wastes generated by countries and their populations can be tracked.
- The quantity of biological resources appropriated for human use is directly related to the amount of bioproductive land area necessary for regeneration and assimilation of waste.
- By weighting each area in proportion to its usable biomass productivity through the use of *equivalence* and *yield* factors, the different areas can be expressed in terms of *global* hectares and added together in a single value.
- Aggregate demand side and nature's supply can be directly compared to each other.

Both footprint and biocapacity accounts include six major types of bioproductive areas used to support human economies [15] that, according to the World Conservation Union classification [16], are cropland, grazing land, forests, fisheries, energy land and built-up land. Each land type is characterized by a specific annual production of usable biomass that can be renewably harvested and is useful to people.

2 METHOD OF CALCULATION

Global Footprint Network, representing a focused group of scientists, academics, governments, businesses and NGOs that utilize the ecological footprint methodology as a resource accounting tool, recommends that *global* hectares are used for footprint and biocapacity assessments unless a compelling reason dictates otherwise. Nevertheless, area demand and supply can be reported in three different ways: *global* hectares (*g*ha), *world-average* hectares (*w*ha), or *nation-specific* actual hectares (*n*ha). A brief clarification is provided here to distinguish among these three units of measure:

- *Global* hectares are needed to measure bioproductivity rather than surface area. Each *global* hectare represents the same fraction of the Earth's total bioproductivity, and is defined as a hectare with world-average productivity for all land types. The ecological production of *global* hectares is calculated by dividing the total ecological production of the Earth by the total biologically productive area available (~11.2 billion hectares). This gives an average productivity per hectare, which is set equal to one *global* hectare. Each *global* hectare can therefore be considered as an average hectare of all land types combined.

  Global hectares provide a useful representation of the ecological demand associated with the flow of a product, as they measure how much of global ecological productivity is required to produce a given flow. They provide more information than simply weight (which does not capture the extent of land and sea area used) or physical area (which does not capture how much ecological production is associated with that land). *Global* hectares are particularly useful for ranking different products based on their total ecological demands and comparing ecological demands associated with products that come from different types of land, such as wheat and wood.
- *World-average* hectares are areas of a specific land type with world-average productivity for that land type (e.g. one hectare of forest land with the ecological production of the average forest hectare globally). *World-average* hectares enable us to visualize the physical extent of demanding a given product if the product were produced on land or water with the world-average productivity. *World-average* hectares are not able to easily compare different land types (it is difficult to compare a world hectare of forest land directly to a world hectare of crop land).

Land of the same type within different countries can also be more easily compared (e.g. demand for timber from Brazil can be compared to demand for timber from Finland using world hectares of forest land).

- *Nation-specific* actual hectares are physical areas of a specific land type located within a specific country and characterized by the bioproductivity of that country. Actual hectares are useful for the visualization of the physical extent occupied by a given activity, but do not communicate information regarding ecological demand. The amount of biological material produced on an actual hectare can vary extremely, depending on its land type and where it is located. For example, a study finding that Italian imports occupy one hectare of Chinese grassland and one hectare of Brazilian forest does not imply that Italy places the same demand on ecosystems in China and Brazil. As Brazilian forests are more productive, occupying one hectare corresponds to a greater demand on the biosphere per hectare. This is equivalent to noting that one actual hectare of Brazilian forest represents more *global* hectares than one actual hectare of Chinese grassland.

Note that all three measures are technically areas expressed in hectares. The prefix '*g*', '*w*' or '*n*' is indicative of a weighted unit, but is not itself a unit. They do not refer to the quantity but to the quality or productivity of the hectare. In fact, ratios between any of these three measures are dimensionless scaling factors. These prefixes can be used as a convenient shorthand, to distinguish among these three different types of productivity-weighted areas.

The use of *yield* and *equivalence* factors allows to readily convert each of these three measures into the others, without any loss of information, as shown in the following equations.

Usually, National Footprint Accounts are reported in a shortened way as the sum of the ratios between local product consumption (or waste generation) and world-average product yield (or absorption factor), for each product (or waste) *i*, multiplied by the appropriate *equivalence* factors. This equation translates a specific area type into average bioproductive area expressed in *global* hectares (*g*ha). For any given nation *n*, the total national footprint is therefore assessed as shown in eqn (1):

$$EF = \sum_i \frac{T_i}{Y_{w_i}} \times EQF_i, \tag{1}$$

where T_i is the annual amount of tonnes (t year^{-1}) of each product *i* that are consumed in the nation *n*; Y_{w_i} is the annual world-average yield $(\text{t ha}^{-1}\ \text{year}^{-1})_{w_i}$ for the production of each product *i*, given by all the annual tonnes of product *i* produced globally, divided by all areas in the world on which this product is grown; EQF_i is the *equivalence* factor for the production of each product *i*.

The *equivalence* factors are evaluated each year for each land category as reported by Wackernagel *et al.* [11]. Each product is currently assigned the equivalence factor for the land type from which it is extracted. They are calculated as the ratio between the maximum potential ecological productivity of world-average land of a specific land type (such as cropland) and the average productivity of all biologically-productive lands on Earth. By showing the productivity difference among land-use categories at the global level, they can be considered land-specific. For example, in 2003, *world-average* cropland was estimated to be 2.2 times more productive than a *world-average* hectare of all biologically productive land and sea area on Earth. Thus, one hectare of *world-average* cropland (*w*ha) was equivalent to 2.2 *global* hectares (*g*ha), and its *equivalence* factor was 2.2 *g*ha *w*ha^{-1}. Since both *g*ha and *w*ha are areas expressed in hectares, they cancel each other, and the *equivalence* factor is technically dimensionless. Therefore, '*g*' and '*w*' prefixes are only used to identify different productivity-weighted areas.

The commonly used shortened eqn (1) hides a fundamental step that can be shown when the equation is expressed in its entirety, as reported in eqn (2):

$$EF = \sum_i \frac{T_i}{Y_{n_i}} \times YF_{n_i} \times EQF_i = \sum_i \frac{T_i}{Y_{n_i}} \times \frac{Y_{n_i}}{Y_{w_i}} \times EQF_i, \tag{2}$$

where Y_{n_i} is the annual nation yield (t ha^{-1} year^{-1})$_{n_i}$ for the production of each product *i* given by all annual tonnes of product *i* produced, divided by all areas in the nation on which this product is grown; YF_{n_i} is the nation-specific yield factor for the production of product *i*.

Yield factors are evaluated annually as the ratio between the yield for the production of each product *i*, in the considered nation, and the yield for the production of that same product in the world as a whole, with the world *yield* factor equal to 1 (by definition). These factors capture the difference between local and global (world-average) productivity within a given land-use category, and are therefore used to convert *nation-specific* (*n*ha) into *world-average* (*w*ha) bioproductive land requirements. For example, suppose an average Italian hectare of cropland annually produces 6 tonnes of wheat, expressed in (t ha^{-1} year^{-1})$_{n_i}$, while the world-average productivity of wheat is 2 (t ha^{-1} year^{-1})$_{w_i}$. As a consequence, the *yield* factor for wheat production is equal to 3. This means that demanding 1 hectare of Italian cropland (*n*ha) is equal to using 3 *world-average* hectares of cropland (*w*ha).

Since eqns (1) and (2) are mathematically identical, world-average yields are currently used in National Footprint Accounts instead of national yield and *yield* factors.

Moreover, by showing the footprint equation as in eqn (2), a more direct linkage with biocapacity accounts is provided. For any given nation, the biocapacity (BC) is assessed by multiplying the land area available annually for production of each product *i*, by the appropriate *yield* and *equivalence* factors as shown in eqn (3).

$$BC = \sum_i A_{n_i} \times YF_{n_i} \times EQF_i, \tag{3}$$

where A_{n_i} represents the estimated bioproductive area expressed in *nation-specific* hectares that is available for the production of each product *i* at the national level. Note that this area is equal to the ratio between the annual amount of tonnes locally produced and the nation average yield of the land producing each product *i*. We refer to this as the production-based specification of the biocapacity equation [17].

As for the footprint assessments, *yield* and *equivalence* factors are used to convert available *nation-specific* actual hectares into *global* hectares.

Both footprint and biocapacity accounting structures, as shown in eqns (2) and (3), have been reported in their entirety to clarify the conversion from *nation-specific* and area-type specific actual hectares to *global* hectares. Other than providing information on the appropriate use of units and scaling factors, the comparison between the two equations shows the theoretical consistency between footprint and biocapacity formulation. In both the equations, nation-specific *yield* factors and land-specific *equivalence* factors are used to convert *nation-specific* and area-type specific actual hectares figures into *global* hectares. In the footprint equation, *nation-specific* and area-type specific actual hectare values represent the bioproductive land area demanded, and are given by the ratio between the annual national consumption of each product *i* and the national yield for each appropriate product. In the biocapacity equation, *nation-specific* and area-type specific actual hectare values are represented by the bioproductive land that is available annually at the national level.

Some analysts have argued that footprint values should be calculated in *global* hectares using national instead of world-average product yield to take into account local resource management, as assumed in eqn (4):

$$EF = \sum_i \frac{T_i}{Y_{n_i}} \times EQF_i. \tag{4}$$

Nevertheless, this calculation is not feasible since, as explained above, the local-related required area obtained (expressed in *n*ha) cannot be multiplied by the appropriate *equivalence* factors, unless *yield* factors are also considered.

Furthermore, as previously shown in eqn (2), it should be noted that the use of national yields and nation-specific *yield* factors for each product *i* corresponds to the use of world-average yield for the same *i*-products.

Nation-specific *yield* factors are indeed the fundamental step necessary for conversion of the bioproductive land requirements from *nation-specific* and area-type specific hectares (*n*ha) to *world-average* area-type specific hectares (*w*ha). This intermediate step finally allows converting *world-average* area-type specific actual (*w*ha) to *global* hectares (*g*ha) by the use of the *equivalence* factors, as described in eqn (2). These two steps are necessary because of the way current *equivalence* factors are calculated [11]. A new method could theoretically be developed to compare, in one single step, *nation-specific* hectares of a given land type directly to *world-average* hectares of all land types, therefore allowing the calculation reported in eqn (4).

3 DISCUSSIONS

The most commonly reported footprint accounting equation (eqn (1)) has led to a lack of clarity in recent years. By accounting for land area requirements using world-average yield, the footprint has been criticized for its supposed inability to deal with local resource management [18]. Even with the understanding that the use of *global* hectares allows for the addition of footprint values of different land categories into a single number and for international comparisons of footprint results of nations [19], it was claimed that this led to a loss of specific local information on the use and management of local natural resources. Hence, many researchers came to the conclusion that the use of a global comparable unit of measure reduces the potential to use footprint for assessing local policies and management criteria [20–23].

It should be noted, in response, that local yields are in fact taken into account since they influence biocapacity evaluations through their effect on the *yield* factor. Potential increases in productivity due to technological or resources management changes are therefore shown as biocapacity increase instead of footprint decrease.

The national footprint values reported in *global* hectares answers the question, 'how much of the planet's regenerative capacity is occupied by a given human activity?'.

At the same time, footprint accounts can also be calculated in *nation-specific* hectares and without applying productivity-based normalization. Footprints expressed in this way answer the question, 'how much physical area is occupied by a given human activity?'.

For projects focused on local resource management, the integrated use of *nation-specific* and *global* hectares may be more appropriate than just *global* hectares [24], while consumption-focused applications that have a more global context may benefit from the use of *global* hectares [25]. While some researchers maintain that only *nation-specific* hectares provide an actual observable measure of demand, others maintain that from a sustainable use perspective, different lands cannot be directly compared or summed without applying some form of weighted productivity [26, 27].

An example of the difference between *nation-specific* actual hectares and *global* hectares is as follows. An Australian whose animals graze on 10 ha of low-productivity grassland will have a footprint far greater than an Italian who consumes the products of 5 ha of very productive cropland when measured in *nation-specific* actual hectares. Measured in *global* hectares, the Italian will have the higher footprint. The most accurate result depends upon the research questions presented above.

4 CONCLUSIONS

The footprint equation is usually shown as shortened for sake of efficiency and consistency with the National Footprint Accounts. This unfortunately has led to misleading interpretations of the method, somehow neglecting the role of the *yield* factors, as well as apparently reducing the consistency between footprint and biocapacity mathematic formulation.

By showing the footprint equation in an expanded form, this article has provided a complete representation of the method reconciling the demand side and the supply side of the accounting structure. Both footprint and biocapacity values are accounted by multiplying *nation-specific* and area-type specific actual land figures, that represent local area requirement and availability, respectively by two 'scaling factors,' that is *yield* and *equivalence* factors. While *yield* factor allows a conversion of the actual area requirement from the national to the global level, the *equivalence* factors are used to express results in terms of *global* hectares.

Global hectares are recommended by Global Footprint Network as a common unit of measure for both footprint and biocapacity assessments under the assumption that: (1) different lands cannot be directly compared or summed without applying some form of productivity weighting, (2) *global* hectares are needed to express footprint and biocapacity results in a cross-country comparable unit of measure. Therefore, *global* hectares have the advantage of expressing both demand and supply of different land areas in single numbers (footprint and biocapacity, respectively) that can be compared to each other.

Nevertheless, reporting footprint and biocapacity results in *global* hectares alone may not be sufficient for analyzing the use and management of local natural resources. On one hand, footprint accounts aimed at a global consistency and cross-country comparability can be reported in *global* hectares. On the other hand, applications aimed at a deep analysis of local environmental burdens and resource management, as well as local production systems, should follow an integrated approach. It may be valuable to report use of *nation-specific* actual hectares beyond the *global* hectares results. As stated in Section 2, the equations presented demonstrate the complete interchangeability of the two types of results obtained when *yield* and *equivalence* factors are used together. Finally, the equations are evidence that local information is embedded in the *global* hectare, and one can be translated into the other. By considering *nation-specific* actual hectares figures, as well as *yield* and *equivalence* factors, footprint and biocapacity accounts capture not only the extension of the locally required or available land areas, but also their biological productivity.

REFERENCES

[1] Rees, W.E., Ecological footprints and appropriated carrying capacity: what urban economics leaves out. *Environment and Urbanization*, **4(2)**, pp. 121–130, 1992.

[2] Wackernagel, M. & Rees, W.E., *Our Ecological Footprint: Reducing Human Impact on the Earth*, New Society Publishers: Gabriola Island, British Columbia, Canada, 1996.

[3] Wackernagel, M., Onisto, L., Linares, A.C., Falfan, I.S.L., Garcia, J.M., Guerrero, A.I.S. & Guerrero, M.G.S., *Ecological Footprint of Nations: How Much Nature Do They Use? How Much Nature Do They Have?* International Council for Local Environment Initiatives, Toronto, Commissioned by the Earth Council for the RIO +5 forum, 1997.

[4] Wackernagel, M., Kitzes, J., Moran, D., Goldfinger, S. & Thomas, M., The ecological footprint of cities and regions: comparing resource availability with resource demand. *Environment and Urbanization*, **18**, pp. 103–112, 2006.

[5] Wackernagel, M., Schulz, N.B., Deumling, D., Linares, A.C., Jenkins, M., Kapos, V., Monfreda, C., Loh, J., Myers, N., Norgaard, R. & Randers, J., Tracking the ecological overshoot of the human economy. *PNAS*, **99(14)**, pp. 9266–9271, 2002.

[6] Chambers, N., Simmons, C. & Wackernagel, M., *Sharing Nature's Interest: Ecological Footprint as an Indicator of Sustainability*. Earthscan Publication Ltd: London and Sterling, VA, 2000.

[7] Folke, C., Jansson, A., Larsson, J. & Costanza, R., Ecosystem appropriation by cities. *Ambio*, **26(3)**, pp. 167–172, 1997.

[8] Lewan, L. & Simmons, C., *The Use of Ecological Footprint and Biocapacity Analyses as Sustainability Indicators for Sub-national Geographical Areas: A Recommended Way Forward. Final Report*, European Common Indicators Project EUROCITIES/Ambiente Italia, 2001.

[9] Monfreda, C., Wackernagel, M. & Deumling, D., Establishing national natural capital accounts based on detailed ecological footprint and biological capacity assessments. *Land Use Policy*, **21**, pp. 231–246, 2004.

[10] Kitzes, J., Peller, A., Goldfinger, S. & Wackernagel, M., Current methods for calculating national ecological footprint accounts. *Science for Environment & Sustainable Society*, **4(1)**, pp. 1–9, 2007.

[11] Wackernagel, M., Monfreda, C., Moran, D., Wermer, P., Goldfinger, S., Deumling, D. & Murray, M., National Footprint and Biocapacity Accounts 2005: the underlying calculation method. Global Footprint Network, Oakland, available at www.footprintnetwork.org, 2005.

[12] Wackernagel, M. & Galli, A., An overview on ecological footprint and sustainable development: a chat with Mathis Wackernagel. *International Journal of Ecodynamics*, **2(1)**, pp. 1–9, 2007.

[13] Odum, H.T., Self-organization, transformity, and information. *Science*, **242**, pp. 1132–1139, 1988.

[14] Tiezzi, E., *Steps Towards an Evolutionary Physics*, WIT Press: Southampton, UK, 2006. Italian version: *Verso una fisica evolutiva*, Donzelli editore: Roma, 2006.

[15] Wackernagel, M., Monfreda, C. & Deumling, D., *Ecological footprint of nations, November 2002 Update: How Much Nature Do They Use? How Much Nature Do They Have?* Redefining Progress, Sustainability Issue Brief, November 2002.

[16] The World Conservation Union, United Nations Environment Programme and the World Wide Fund for Nature, *Caring for the Earth: A Strategy for Living Sustainably*, IUCN, UNEP and WWF: Gland, Switzerland, 1991.

[17] Kitzes, J., Galli, A., Wackernagel, M., Goldfinger, S. & Bastianoni, S., A 'constant global hectare' method for representing ecological footprint time trends. Paper submitted to the *International Ecological Footprint Conference – Stepping Up the Pace: New Developments in Ecological Footprint Methodology, Policy & Practice*, 8–10 May, Cardiff, 2007.

[18] Luck, M.A., Darrel Jenerette, G., Wu, J. & Grimm, N.B., The urban funnel model and the spatially heterogeneous ecological footprint. *Ecosystems*, **4**, pp. 782–796, 2001.

[19] Ferguson, A.R.B., The logical foundations of ecological footprints. *Environment, Development and Sustainability*, **1**, pp. 149–156, 1999.

[20] van Vuuren, D.P. & Smeets, E.M.W., Ecological footprints of Benin, Bhutan, Costa Rica and the Netherlands. *Ecological Economics*, **34**, pp. 115–130, 2000.

[21] Krausmann, F., Land use and industrial modernization: an empirical analysis of human influence on the functioning of ecosystems in Austria 1830–1995. *Land Use Policy*, **18**, pp. 17–26, 2001.

[22] Krausmann, F., Haberl, H., Schulz, N.B., Erb, K.H., Darge, E. & Gaube, V., Land-use change and socio-economic metabolism in Austria – Part I: Driving forces of land-use change: 1950–1995. *Land Use Policy*, **20**, pp. 1–20, 2003.

[23] Haberl, H., Erb, K.H., Krausmann, F., Adensam, H. & Schulz, N.B., Land-use change and socio-economic metabolism in Austria – Part II: Land-use scenarios for 2020. *Land Use Policy*, **20**, pp. 21–39, 2003.

[24] Wiedmann, T. & Lenzen, M., On the conversion between local and global hectares in ecological footprint analysis. *Ecological Economics*, **60**, pp. 673–677, 2007.

[25] Wackernagel, M., Monfreda, C., Schulz, N.B., Erb, K.H., Haberl, H. & Krausmann, F., Calculating national and global ecological footprint time series: resolving conceptual challenges. *Land Use Policy*, **21**, pp. 271–278, 2004.

[26] Venetoulis, J. & Talberth, J., Refining the ecological footprint. *Environment, Development and Sustainability*, DOI 10.1007/s10668-006-9074-z, 2007.

[27] Kitzes, J., Galli, A., Bagliani, M., Barrett, J., Dige, G., Ede, S., Erb, K., Giljum, S., Haberl, H., Hails, C., Jungwirth, S., Lenzen, M., Lewis, K., Loh, J., Marchettini, N., Messinger, H., Milne, K., Moles, R., Monfreda, C., Moran, D., Nakano, K., Pyhälä, A., Rees, W., Simmons, C., Wackernagel, M., Wada, Y., Walsh, C. & Wiedmann, T., A research agenda for improving national ecological footprint accounts. Paper submitted to the *International Ecological Footprint Conference – Stepping Up the Pace: New Developments in Ecological Footprint Methodology, Policy & Practice*, 8–10 May, Cardiff, 2007.